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Quantum Forms of Tensor Products

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Abstract. We describe the skew tensor-product structure inherent to Yang-Mills procedures pertaining to non-commutative spaces obtained by (skew) tensorization.

0. Introduction

In the fall of 1991 Alain Connes introduced [2] the differential algebra of *quantum differential forms* (non-commutative generalization of the classical De Rham complex) and used this concept to obtain an improved form of his non-commutative Yang-Mills scheme, in particular as applied to the standard model of elementary particles. The improvement resides in the removal of the cumbersome adynamical fields which plagued the previous formalism (a price to be paid for the use of formal differential forms instead of the genuine quantum differential forms). We refer our reader to ref. [2] versus ref. [1] – or, for more details, to [3] III versus [3] I and II, for a comparison of the two methods.

Let $\mathbf{A}$ be a (generally non-commutative, possibly $\mathbf{Z}/2$-graded) associative $*$-algebra endowed with the (non-commutative) riemannian geometry specified by a D-summable K-cycle (H, D) [2]. The *generalized De Rham complex* (or *set of D-quantum forms*) $\Omega_D\mathbf{A}$ of $\mathbf{A}$ if the differential algebra, with differential δ, quotient of the unital differential envelope $(\Omega\mathbf{A}, \delta)$ of $\mathbf{A}$ (cf. e.g. [4]) by the graded ideal $K^* + \delta K^*$, $K^* = \bigoplus_{n\in\mathbf{N}} \operatorname{Ker} \pi_D \cap \Omega\mathbf{A}^n$, homogenized kernel of the representation π_D of $\Omega\mathbf{A}$ specified by D (cf. [2]).

In the application to the standard model, the (electro-weak side of) the algebra is the tensor product $\mathbf{A} = C^\infty(\mathbf{M}) \otimes (\mathbf{C} \oplus \mathbf{H})$ of the algebra of smooth functions on the space-time spin^c riemannian manifold $\mathbf{M}$ by the algebra $\mathbf{C} \oplus \mathbf{H}$ embodying the $U(1) \times SU(2)$ (innerspace-) degrees of freedom. The K-cycle D is the tensor product (see below (16)) of the Dirac K-cycle of $C^\infty(\mathbf{M})$ by an inner-space K-cycle (H_f, D_f) specified by the mass matrix of the fermions. This tensor product structure raises the question of how the differential algebra $\Omega_D\mathbf{A}$ of electro-weak quantum forms relates