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The Automorphism Group of the Irrational Rotation C*-Algebra

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Abstract. The structure of the automorphism group of a simple C*-algebra of real rank zero which is an inductive limit of circle algebras is described. In particular, it is proved that the automorphism group of the irrational rotation C*-algebra, A_θ, for any irrational number θ, is an extension of a topologically simple group by $\mathrm{GL}_2(\mathbb{Z})$.

1. Introduction

Let A be a unital C*-algebra. The automorphism group $\mathrm{Aut}(A)$ of A decomposes into a series

$$\overline{\mathrm{Inn}}_0(A) \lhd \overline{\mathrm{Inn}}(A) \lhd \mathrm{Aut}(A)\,,$$

where $\overline{\mathrm{Inn}}(A)$ is the group of approximately inner automorphisms, and $\overline{\mathrm{Inn}}_0(A)$ is the closure of the group of inner automorphisms determined by unitaries connected to 1.

We shall prove, using an argument closely following a paper by de la Harpe and Skandalis, [HS], that if A is a simple C*-algebra of real rank zero satisfying some extra conditions, then the group $\overline{\mathrm{Inn}}_0(A)$ is topologically simple (Corollary 2.4). In particular, $\overline{\mathrm{Inn}}_0(A)$ is topologically simple for all simple inductive limits of circle algebras which have real rank zero. It seems likely that $\overline{\mathrm{Inn}}_0(A)$ is topologically simple for all simple C*-algebras.

By the classification theorem for inductive limits of circle algebras of real rank zero, [E2], it follows that the quotient group $\mathrm{Aut}(A)/\overline{\mathrm{Inn}}(A)$ is isomorphic to the group of automorphisms of the K-theory of A (Theorem 2.1).

Sections 3 and 4 are concerned with computing the quotient group $\overline{\mathrm{Inn}}(A)/\overline{\mathrm{Inn}}_0(A)$ for a C*-algebra A which is a simple inductive limit of circle