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Generalized Drinfeld-Sokolov Reductions and KdV Type Hierarchies

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Abstract. Generalized Drinfeld-Sokolov (DS) hierarchies are constructed through local reductions of Hamiltonian flows generated by monodromy invariants on the dual of a loop algebra. Following earlier work of De Groot et al., reductions based upon graded regular elements of arbitrary Heisenberg subalgebras are considered. We show that, in the case of the nontwisted loop algebra $\ell(gl_n)$, graded regular elements exist only in those Heisenberg subalgebras which correspond either to the partitions of n into the sum of equal numbers $n = pr$ or to equal numbers plus one $n = pr + 1$. We prove that the reduction belonging to the grade 1 regular elements in the case $n = pr$ yields the $p \times p$ matrix version of the Gelfand-Dickey r-KdV hierarchy, generalizing the scalar case $p = 1$ considered by DS. The methods of DS are utilized throughout the analysis, but formulating the reduction entirely within the Hamiltonian framework provided by the classical r-matrix approach leads to some simplifications even for $p = 1$.

0. Introduction

The generalized KdV type hierarchies of Drinfeld and Sokolov (DS) are among the most important examples in the field of integrable evolution equations [1]. They also play an important rôle in current studies of two-dimensional gravity [2] and in conformal field theory [3]. The "second Gelfand-Dickey" Poisson bracket of these bihamiltonian systems is a reduction of the affine current algebra Lie-Poisson bracket, and it gives an extension of the Virasoro algebra by conformal tensors. Such extended conformal algebras are called $\mathscr{W}$-algebras and have received a lot of attention recently [4–6].

The motivation for the present work was to gain, from a purely Hamiltonian viewpoint, a better understanding of the reduction procedure used in [1] and the generalizations proposed in a recent series of papers [7–9] aimed at the construction of new integrable hierarchies and $\mathscr{W}$-algebras.

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