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On Sums of q-Independent $SU_q(2)$ Quantum Variables

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Abstract. A representation-free approach to the q-analog of the quantum central limit theorem for $\mathscr{E} = SU_q(2)$ is presented. It is shown that for certain functionals $\phi \in \mathscr{E}^*$ one can derive a version of a quantum central limit theorem (qclt) with $\sqrt{[N]}$ as a scaling parameter, which may be viewed as a q-analog of qclt.

1. Introduction

Limit theorems in quantum probability are related to the notion of independence. Depending on its kind we obtain various approaches to quantum limit theorems, in particular quantum central limit theorems (qclt).

The study of qclt's originated with the works of Giri and Waldenfels [4] for commuting independence and Waldenfels [11] for anticommuting independence. Those works gave boson and fermion versions of qclt. A general approach for coalgebras with independence introduced through the coproduct was presented by Schürmann [7]. In [2] Accardi and Lu proved a qclt for weakly dependent maps.

Voiculescu [10] developed a general theory for free products (free independence). Following his ideas, Speicher [9] proved a general limit theorem giving the free analogues of Gaussian and Poisson distributions. A q-example of Brownian was considered by Bożejko and Speicher [3].

Recently, a q-version of quantum central limit theorem (q-independence) and a q-version of white noise was presented by Schürmann [8]. His qclt was based on the qclt for coalgebras. He assumed that ϕ agrees with δ on $\mathscr{E}^{(0)}$, where $\mathscr{E} = \mathscr{E}^{(0)} \oplus \mathscr{E}^{(1)} \oplus \dots$ is a $\mathbf{N}$-graduation on $\mathscr{E}$ that is compatible with the coproduct and δ is a counit. Independently, in [6] we studied a q-analog of qclt for $SU_q(2)$ for q real positive. Our approach was group-theoretic and related to a certain group contraction of $SU_q(2)$. In our work the scaling q-qclt constant was not $\sqrt{N}$ but $\sqrt{[N]}$, where $[N]$ is the

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