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# Singularity, Complexity, and Quasi-Integrability of Rational Mappings

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**Abstract.** We investigate global properties of the mappings entering the description of symmetries of integrable spin and vertex models, by exploiting their nature of birational transformations of projective spaces. We give an algorithmic analysis of the structure of invariants of such mappings. We discuss some characteristic conditions for their (quasi)-integrability, and in particular its links with their singularities (in the 2-plane). Finally, we describe some of their properties *qua* dynamical systems, making contact with Arnol'd's notion of complexity, and exemplify remarkable behaviours.

## 1. Introduction

We want to analyze in detail some realizations of Coxeter groups [1, 2] by birational transformations of projective spaces which have been shown to appear in the description of the symmetries of quantum integrable systems [3–5, 6–10].

The first motivation to look at these realizations resides of course in their relations with the star-triangle and the Yang-Baxter equations or their higher dimensional generalizations such as the tetrahedron equations. A characteristic feature of the orbits of the known solutions of the Yang-Baxter equations under these groups is that they are confined to subvarieties of high codimension of the parameter space (actually curves), *signaling the existence of an unexpectedly large number of algebraically independent invariants*. The discovery and the analysis of the possible invariants is a decisive step in the study of the Yang-Baxter (tetrahedron,...) equations, in particular for what concerns the so-called baxterization problem [11].

Another motivation is to use these realizations to construct discrete time evolution maps, as it is usual in the study *à la* Poincaré of dynamical systems, by iterating some element of the group. One of the main questions in this setting is again to bring to light the possible presence of invariants and invariant tori [12–14, 15–19, 6–10].

If a realization admits algebraic invariants, we will say it has a property of quasi-integrability.

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