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Vortices and Localization in Euler Flows*

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Abstract. We study the time evolution of a non-viscous incompressible two-dimensional fluid when the initial vorticity is concentrated in N small disjoint regions of diameter ε. We prove that the time evolved vorticity is also concentrated in N regions of diameter d, vanishing as $\varepsilon \to 0$. As a consequence we give a rigorous proof of the validity of the point vortex system. The same problem is examined in the context of the vortex-wave system.

1. Introduction

This paper is devoted to the study of the behavior of the time evolution of a non-viscous incompressible two-dimensional fluid, when the initial data becomes singular. Namely we study the case in which the initial vorticity is sharply concentrated in N small disjoint regions of diameter ε. We prove that the time evolved vorticity is also concentrated in N small regions. More precisely we prove that, with the total vorticity of each region fixed and an arbitrary time $t > 0$, the support of the vorticity is contained in N disjoint disks of radius d, d vanishing with ε. We call this property "localization."

The difficulty in proving this localization property relies in the divergent kernel describing the interaction among the vorticity elements. Actually, when ε is very small, the velocity field in each blob becomes very large and it is difficult to exclude that the radial component of the velocity pushes away thin filaments of vorticity. We prove that this does not happen.

At the same time, as a main consequence of the present result, we prove a general rigorous connection between the Euler Equation and the point vortex theory (for the first definition of the point vortex system see [1], for a review on the topic see [2, 3]).

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