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Geometry and Kinematics of Two Skyrmions

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Abstract. In Skyrme's soliton model of baryons, a single Skyrmion has six degrees of freedom, so it is expected that two-Skyrmion dynamics at modest energies can be modelled by motion on a 12-dimensional space of Skyrme fields. A candidate for this space is generated by the gradient flow of the potential energy function, descending from the unstable, baryon number two, hedgehog solutions of the Skyrme field equation. An apparently very similar space is obtained by restricting the gradient flow to the Skyrme fields derived from $SU(2)$ Yang-Mills instantons of charge two. On both of these spaces, one may quotient out by the group of translations and isospin rotations. Hartshorne's geometrical description of charge two instantons leads us to a conjecture for the global structure of the 6-dimensional quotient space. The conjectured structure is that of complex projective 3-space, with complex conjugate points on one projective plane identified and the real points in this plane removed.

1. Introduction

A fundamental problem in non-integrable soliton models of elementary particles is to find an approximate description of soliton interactions in which each soliton has the same number of degrees of freedom as a single isolated soliton. Such a description reduces a field theory with infinitely many degrees of freedom to a "particle" theory, and is appropriate at modest energies where radiation and particle-antiparticle pair production can be neglected. In certain special Lagrangian field theories, there is a moduli space of exact static solutions of a Bogomol'nyi-type equation [1], representing multi-soliton configurations, and the dimension of the k-soliton moduli space, suitably defined, is of the form Nk, where N is the number of degrees of freedom of a single soliton. For $SU(2)$ Yang-Mills-Higgs monopoles, for example, N is four [2], and for vortices in the abelian Higgs model, N is two [3]. The moduli space is a flat valley of minimal energy in the configuration space of fields at a given time, and has a natural Riemannian metric inherited from the kinetic terms of the field theory Lagrangian. The classical dynamics of the solitons is modelled by geodesic motion on the moduli