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Classification of Generic 3-dimensional Lagrangian Singularities with $(\mathbf{Z}_2)^l$-symmetries

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Abstract. The paper provides the complete list of local models for $\mathbf{Z}_2^l$-invariant generic germs of Lagrangian submanifolds of dimension ≤ 3. Classification is done directly for genrating functions of Lagrangian submanifolds and contains both elementary singularities and non-elementary ones with continuous moduli. The results demonstrate, in particular, that in contrast to the non-equivariant case the classification of equivariant Lagrangian singularities is not subordinated to the classification of symmetric functions up to the right equivariant equivalences.

1. Introduction

One of the most important steps in the initial development of singularity theory of Lagrangian submanifolds was finding that the singularities of (*non-equivariant*) canonical Lagrangian projections are completely determined (at least locally) by singularities of smooth generating functions (or generating families of functions). A crucial contribution to the problem was made by Arnold [2] who found the complete classification of stable singularities of Lagrangian submanifolds of dimension ≤ 5, inspiring further investigations in that direction (cf. [4, 3, 8, 25]). The standard (non-symmetric) theory of Lagrangian singularities has various important applications. In many of them non-trivial symmetries appear as an additional constraint and thus the problem of classification of $\mathcal{G}$-invariant Lagrangian submanifolds (with $\mathcal{G}$ being a compact Lie group of symmetry) emerges naturally. This problem was introduced and initially investigated in [13], then the formal stability theory was continued in the papers [15, 16].

In the present paper, we investigate the discrete groups of symmetries $\mathbf{Z}_2^l$. Such symmetries appear for instance in the problem of determination of symmetric caustics in geometrical optics of lenses [5, 17], in thermodynamical phase transitions in ordered systems [15] and in equivariant bifurcation theory [10]. The considerations are more complex and technical than in the non-equivariant case. On top of the usual complications caused by the symmetry we encounter additional obstacles. Firstly,