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The Lie-Poisson Structure of Integrable Classical Non-Linear Sigma Models

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Abstract. The canonical structure of classical non-linear sigma models on Riemannian symmetric spaces, which constitute the most general class of classical non-linear sigma models known to be integrable, is shown to be governed by a fundamental Poisson bracket relation that fits into the $r - s$-matrix formalism for non-ultralocal integrable models first discussed by Maillet. The matrices r and s are computed explicitly and, being field dependent, satisfy fundamental Poisson bracket relations of their own, which can be expressed in terms of a new numerical matrix c. It is proposed that all these Poisson brackets taken together are representation conditions for a new kind of algebra which, for this class of models, replaces the classical Yang-Baxter algebra governing the canonical structure of ultralocal models. The Poisson brackets for the transition matrices are also computed, and the notorious regularization problem associated with the definition of the Poisson brackets for the monodromy matrices is discussed.

1. Introduction

During the last two decades, there has been great progress in understanding the structure of two-dimensional integrable field theories. Within the Hamiltonian approach [6], there have emerged new algebraic structures, notably the concept of Yang-Baxter algebras. These algebras appear, e.g., through the commutation relations of monodromy matrices when solving the models by the inverse scattering method.

At the classical level one begins by rewriting the equations of motion as a zero curvature condition, i.e., as the compatibility condition for a linear system with spectral parameter (Lax pair). In the Hamiltonian context one then studies the Poisson brackets

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