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A Uniqueness Condition for Gibbs Measures, with Application to the 2-Dimensional Ising Antiferromagnet

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Abstract. A uniqueness condition for Gibbs measures is given. This condition is stated in terms of (absence of) a certain type of percolation involving two independent realisations. This result can be applied in certain concrete situations by comparison with "ordinary" percolation. In this way we prove that the Ising antiferromagnet on a square lattice has a unique Gibbs measure if $\beta(4 - |h|) < \frac{1}{2}\ \ln(P_c/(1 - P_c))$, where h denotes the external magnetic field, β the inverse temperature, and P_c the critical probability for site percolation on that lattice. Since P_c is larger than $\frac{1}{2}$, this extends a result by Dobrushin, Kolafa and Shlosman (whose proof was computer-assisted).

1. Introduction and General Theorem

Our main theorem requires hardly any prerequisites and we hope the following introduction makes it also accessible to non-experts.

Let the graph G be connected, countably infinite, and locally finite (the last means that each vertex has finitely many edges). The set of vertices of G is denoted by V_G. Vertices will typically be denoted by i, j, v, w etc., possibly with a subscript. Two vertices v and w are said to be *adjacent*, or *neighbours* (notation: $v \sim w$) if there is an edge between them.

A *path* from v to w is a sequence of vertices $v_1 = v, v_2, \ldots, v_l = w$ with the property that consecutive vertices are adjacent. An infinite path is a sequence $v_1, v_2, \ldots$ with the property that consecutive vertices are adjacent, and which contains infinitely many different vertices.

For $B \subset V_G$, δ_B will denote the *boundary* of B, i.e. the set of all vertices which are not in B but adjacent to some vertex in B.

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