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Fusion and Singular Vectors in $A_1^{(1)}$ Highest Weight Cyclic Modules

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Abstract. We show how the interplay between the fusion formalism of conformal field theory and the Knizhnik-Zamolodchikov equation leads to explicit formulae for the singular vectors in the highest weight representations of $A_1^{(1)}$.

I. Introduction

Infinite dimensional Lie algebras occur everywhere in the study of 2-d conformal field therories: the Virasoro algebra and the affine algebras are the most common examples. However, the construction of the irreducible representations of these algebras is quite involved. Singular vectors are important because they indicate the existence of subrepresentations in a given representation. In the affine case, Kac and Kazhdan [12] gave the criterion for the reducibility or irreducibility of the Verma modules and Malikov, Feigin, and Fuks [16] found a formula for the singular vectors. This formula looks very simple, but involves an analytic continuation to make sense, which makes it very difficult to use.

Apart from the purely mathematical description, several approaches motivated by physics have been proposed, based on vertex operators (see [18] for a general reference dealing with $A_1^{(1)}$), bosonization and variants of the Feigin and Fuks construction and BRST cohomology [4]. In the physical context, the importance of singular vectors comes from Ward identities: to calculate a correlation function involving a descendent of a primary field, one simply applies a linear operator to the correlation function of the primary [2]. A singular vector is a descendent that is set to zero in an irreducible representation, with the consequence that the correlation functions of the corresponding primary satisfy closed linear relations, leading to a contour integral representation.

One of the aims of this paper is to show that elementary methods of conformal field theory allow us to understand some important features of the structure of

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