Commun. Math. Phys. 152, 41–71 (1993)

Communications in Mathematical Physics

$N=2$ Topological Gauge Theory, the Euler Characteristic of Moduli Spaces, and the Casson Invariant

Matthias Blau[1,3] **and George Thompson**[2,3]

[1] NIKHEF-H, P.O. Box 41882, 1009 DB Amsterdam, The Netherlands
[2] Institut für Physik, Johannes-Gutenberg-Universität Mainz, Staudinger Weg 7, W-6500 Mainz, Germany

Received December 24, 1991; in revised form August 11, 1992

Abstract. We discuss gauge theory with a topological $N=2$ symmetry. This theory captures the de Rham complex and Riemannian geometry of some underlying moduli space $\mathcal{M}$ and the partition function equals the Euler number $\chi(\mathcal{M})$ of $\mathcal{M}$. We explicitly deal with moduli spaces of instantons and of flat connections in two and three dimensions. To motivate our constructions we explain the relation between the Mathai-Quillen formalism and supersymmetric quantum mechanics and introduce a new kind of supersymmetric quantum mechanics based on the Gauss-Codazzi equations. We interpret the gauge theory actions from the Atiyah-Jeffrey point of view and relate them to supersymmetric quantum mechanics on spaces of connections. As a consequence of these considerations we propose the Euler number $\chi(\mathcal{M})$ of the moduli space of flat connections as a generalization to arbitrary three-manifolds of the Casson invariant. We also comment on the possibility of constructing a topological version of the Penner matrix model.

Contents

[1] e-mail: blau@ictp.trieste.it
[2] e-mail: thompson@ictp.trieste.it
[3] From Oct. 1992: ictp, P.O. Box 586, I-34100 Trieste, Italy