Commun. Math. Phys. 151, 531–542 (1993)

Communications in
Mathematical
Physics

Equivalence of Dirac and Intrinsic Quantization for Non-free Group Actions

C. Emmrich
Fakultät für Physik der Universität Freiburg, Hermann-Herder-Str. 3 D-7800 Freiburg, FRG

Received April 2, 1992

Abstract. In this article we generalize some results on the equivalence of Dirac quantization and intrinsic quantization proven in [3]: We consider systems with first class constraints that may be considered as the vanishing of the momentum map to a lifted group action, but drop the assumption that the group action is free as well as the assumption that the group is compact. Using a generalized Weyl ordering prescription applicable to arbitrary cotangent bundles we derive necessary and sufficient conditions for the equivalence of the two approaches for different classes of functions analogous to those for the free case, although the proofs given in [3] must be considerably modified and refined due to the non-compactness of the orbits and the lack of sufficiently many invariant vector fields. The same strong obstruction as in the free case is found if one requires equivalence for all invariant functions, essentially only admitting trivial bundles.

1. Introduction

The different approaches to the quantization of systems with first class constraints, namely extrinsic (in particular Dirac) quantization, which first quantizes the unconstrained system and then imposes the constraints as conditions on the states, and intrinsic quantization, which first classically eliminates the constraints and then quantizes the resulting unconstrained system, are known not to be equivalent in general. However, for the case that the constraints consist in the vanishing of the canonical momentum map belonging to the lift of a group action on configuration space, some general positive results have been proven in the literature concerning the existence of a natural isomorphism between the Hilbert spaces which intertwines the operators corresponding to observables at most linear in the momenta ([4–6, 3] and references cited therein).

In [3] we have proven for the case of the free action of a compact group that for a reasonable quantization scheme this natural isomorphism of Hilbert spaces intertwines the operators corresponding to observables of higher order in the momenta only under additional, rather restrictive conditions. In these proofs the