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Holonomy Groups and W-Symmetries

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Abstract. Irreducible sigma models, i.e. those for which the partition function does not factorise, are defined on Riemannian spaces with irreducible holonomy groups. These special geometries are characterised by the existence of covariantly constant forms which in turn give rise to symmetries of the supersymmetric sigma model actions. The Poisson bracket algebra of the corresponding currents is a W-algebra. Extended supersymmetries arise as special cases.

1. Introduction

It has been known for many years that the geometry of the target space of two dimensional supersymmetric sigma models is restricted when there are further supersymmetries; in particular, $N = 2$ supersymmetry requires that the target space be a Kähler manifold [1], and $N = 4$ supersymmetry requires that it be a hyperkähler manifold [2]. More exotic geometries arise in heterotic sigma models with torsion and in one-dimensional models [3, 4, 9]. More recently it has been realised that sigma models can admit further symmetries which are non-linear in the derivatives of the sigma model field. The prototype of this type of symmetry is the non-linear realisation of supersymmetry using free fermions [5]; further instances have been given in the context of supersymmetric particle mechanics [6, 7] and in $N = 2$ two-dimensional models, where it has been realised that it is not necessary to impose the vanishing of the Nijenhuis tensor [9, 10]. In [8] a preliminary investigation into non-linear symmetries of other two-dimensional supersymmetric sigma models was presented. A related type of symmetry occurs in bosonic sigma models, the so-called W-symmetry [12, 13].

In this article we combine the issues of the geometry of the target spaces and the non-linear symmetries of two dimensional supersymmetric sigma models. In the case of $N = 2$ and $N = 4$ supersymmetries, for example, the additional structures on the (Riemannian) target spaces reduce the holonomy groups from $O(n)$ to $U\left(\frac{n}{2}\right)$ and $Sp\left(\frac{n}{4}\right)$ respectively, where $n = \dim M$, M being the target space. We