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The Integrability Criterion in SU(2) Chern-Simons Gauge Theory

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Abstract. We prove that the multiplicity spaces appearing in Chern–Simons theory, as defined by Segal, vanish unless they are associated to integrable representations. This and other links with conformal field theory are examined.

1. Introduction

The purpose of this paper is to prove a conjecture attributed to Segal concerning the vanishing of certain multiplicity spaces appearing in the geometric quantization of SU(2) Chern–Simons gauge theory (see [Wi]). The result is closely related to the fact that only the *integrable* representations of the loop group of SU(2) play a role in the theory. The method used in this paper is the analytic description of certain moduli spaces of vector bundles developed in [D-W1].

Let us begin by describing the main result. Throughout the paper, let $\bar{\Sigma}$ denote a compact Riemann surface of genus $g > 3$; let p be a distinguished point of $\bar{\Sigma}$, $\Sigma = \bar{\Sigma} \setminus \{p\}$, and fix $\mathbf{G} = \mathrm{SU}(2)$. We shall denote by $\mathscr{A}_s$ the stable, smooth connections on a trivial $\mathbf{G}$-bundle over $\bar{\Sigma}$, and by $\Delta \to \mathscr{A}_s$ we shall mean the determinant line bundle. For any integer $k \geqq 0$, let $H^0(\mathscr{A}_s, \Delta^{\otimes k})$ denote the infinite dimensional space of holomorphic sections of $\Delta^{\otimes k}$. For λ a non-negative half-integer, let V_λ denote the irreducible representation of $\mathbf{G}$ of dimension $2\lambda + 1$. The complex gauge group $\mathscr{G}^{\mathbb{C}}$ acts on the determinant bundle, and also on V_λ via evaluation at the point p. Following Segal, we define the *space of states*

$$\mathscr{V}_\lambda = \mathrm{Hom}_{\mathscr{G}^{\mathbb{C}}}(V_\lambda^*, H^0(\mathscr{A}_s, \Delta^{\otimes k}))$$

(see [Ox]), where the homomorphisms are required to intertwine the actions of $\mathscr{G}^{\mathbb{C}}$. In Sect. 3 we shall prove the

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