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Selberg Super-Trace Formula for Super Riemann Surfaces II: Elliptic and Parabolic Conjugacy Classes, and Selberg Super-Zeta Functions

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Abstract. Further contributions developing a super analogue of the classical Selberg trace formula, the Selberg super-trace formula, are presented. This paper deals with the calculation of contributions arising from elliptic and parabolic conjugacy classes to the Selberg super-trace formula for super Riemann surfaces. Analytic properties and the functional equation for the corresponding Selberg super-zeta function R_0, R_1 and Z_S, respectively, are derived and discussed. In particular, the elliptic contributions to a super Fuchsian group only alter the multiplicities of the "trivial" zeros and poles of the Selberg super-zeta function R_0, R_1 and Z_S, respectively, already due to the hyperbolic conjugacy classes. The parabolic conjugacy classes introduce new features in the analytical structure.

I. Introduction

In this paper I want to present further contributions to develop a super analogue to the classical Selberg trace formula. This includes also an investigation of the analytic properties of the Selberg super-zeta functions.

Trace formulae emerge in various fields of mathematics and physics, in the latter particularly in the study of fundamental forces. The original version of the Selberg trace formula by Selberg [55] has come from the intention to study number theoretical problems. Actually the Selberg trace formula has some striking similarities with the Weil formula [66] and there is in fact a close relation between the areas of analytic number theory, eigenvalues on compact Riemann surfaces and the Selberg trace formula (e.g. [33]). Of particular interest in all these studies are the analytic properties (zeros and poles) of the Selberg zeta-function $Z(s)$.

There are other reasons as well for studying trace formulae in particular in physics. The Selberg trace formula is a special case of a general class of trace

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