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Bohr-Sommerfeld Orbits in the Moduli Space of Flat Connections and the Verlinde Dimension Formula*

Lisa C. Jeffrey[1,**] **and Jonathan Weitsman**[2,***]

[1] School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA
[2] Department of Mathematics, University of California, Berkeley, CA 94720, USA

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Abstract. We show how the moduli space of flat $SU(2)$ connections on a two-manifold can be quantized in the real polarization of [15], using the methods of [6]. The dimension of the quantization, given by the number of integral fibres of the polarization, matches the Verlinde formula, which is known to give the dimension of the quantization of this space in a Kähler polarization.

1. Introduction

Let Σ^g be a (compact, oriented) two-manifold of genus g, and consider the moduli space $\bar{\mathscr{S}}_g$ of flat $SU(2)$ connections on Σ^g. This space contains a large open set $\mathscr{S}_g$ which is a symplectic manifold with symplectic form ω such that $2\pi i\omega$ is the curvature of a natural line bundle $\mathscr{L}$ on $\bar{\mathscr{S}}_g$. The quantization of this prequantum system has been the subject of much recent interest. Much of the mathematical work on this topic has concentrated on the Verlinde formula for the dimension of the quantization in a Kähler polarization.

In [15, 16] there was introduced a different approach to the quantization procedure, based on a real polarization of the space $\bar{\mathscr{S}}_g$. If (M,ω) is a compact symplectic manifold of dimension $2m$, a real polarization of M is a map $\pi: M \to B$ onto a manifold B of dimension m, such that $\omega|_{\pi^{-1}(b)}=0$ for all $b\in B$. Under sufficiently strong hypotheses, a submanifold L appearing as a fibre $\pi^{-1}(b)$ must be a torus of dimension m, and the quantization procedure for a prequantum system over (M,ω) given by a line bundle $\mathscr{L}\to M$ with connection of curvature $2\pi i\omega$ is particularly simple. For there will be a finite number of fibres L_i of the

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