Global Existence of Solutions of the Spherically Symmetric Vlasov–Einstein System with Small Initial Data

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Abstract. We show that global asymptotically flat singularity-free solutions of the spherically symmetric Vlasov–Einstein system exist for all initial data which are sufficiently small in an appropriate sense. At the same time detailed information is obtained concerning the asymptotic behaviour of these solutions. A key element of the proof which is also of intrinsic interest is a local existence theorem with a continuation criterion which says that a solution cannot cease to exist as long as the maximum momentum in the support of the distribution function remains bounded. These results are contrasted with known theorems on spherically symmetric dust solutions.

1. Introduction

The Vlasov–Einstein system describes a self-gravitating collisionless gas within the framework of general relativity. The only available theorem on the initial value problem for this system is a local in time existence result due to Choquet–Bruhat [3]. There are two reasons why it is of interest to study this initial value problem. In recent years there has been considerable progress in understanding the closely related Vlasov–Poisson and Vlasov–Maxwell systems and similar equations. In particular, it has been shown by Pfaffelmoser [19] (see also [17, 23]) that the Vlasov–Poisson system, which is the non-relativistic analogue of the Vlasov–Einstein system, has global classical solutions for general initial data. In the case of the Vlasov–Maxwell system the corresponding question is still open but a number of partial results have been obtained. (For details see [20].) Since the Vlasov–Einstein system, which up to now has been studied very little, has deep relations to these other equations, one motivation for studying it is to obtain a better understanding of the initial value problem for a whole class of differential equations which are of intrinsic mathematical interest and have varied physical applications. The other motivation is that the Vlasov equation provides a matter model for general relativity which seems particularly suitable for the study of the long-time behaviour of matter in gravitational fields.