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Quantum Mechanics of Gravitational Collapse

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Abstract. A toy model of gravitational collapse in General Relativity is studied. It consists of a spherically symmetric thin shell of dust with a fixed rest mass. The configuration space is the half-axis and the Hamiltonian splits into a differential operator of infinite order ("free" Hamiltonian) and a "Coulomb" potential. Harmonic analysis on the half-axis is used to define the free Hamiltonian. For rest masses comparable to, or lower than one Planck mass, the Kato–Rellich theorem is applicable and one self-adjoint extension of the full Hamiltonian is found. A boundary condition for the wave function results whose effect is to keep the shell away from the singularity. This will lead to superposition of states containing both black and white holes.

1. Introduction and Summary

The classical theory of gravity, General Relativity, suffers from the problem of singularities. Gravitational collapse cannot be halted and it leads generically to infinite densities and curvatures. This divergence contradicts the basic postulates of the theory (e.g., locally Minkowskian spacetime, see e.g. [7]). A possible solution of this problem is often sought in quantum theory. However, attempts to construct quantum gravity have not been successful as yet.

In the present paper, we try to circumvent the construction of quantum gravity by working with a more tractable model, which still suffers in its classical form from the same disease. The model we choose is a spherically symmetric thin shell of dust with a fixed rest mass M and its gravitational field as given by Einstein's equations. Thin shells have become quite popular as models for various phenomena in recent years (e.g. Refs. [1, 3, 15, and 19]).

In principle, there are at least two objections against the use of such minisuperspace models. First, suppressing most of the degrees of freedom can destroy some property of the original system which is relevant for the problem under study (cf. [12]). Thus, in our case, the result that there is no thermodynamics may be due to the freezing of all but one degree of freedom (see later). Second, canonical reduction of the degrees of freedom to just the physical ones needs a gauge fixing which, for