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The Parallel Propagator as Basic Variable for Yang–Mills Theory

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Abstract. The parallel propagator (associated with a Yang–Mills connection) taken along all null geodesics from a field point x to null infinity is introduced as a basic variable in Yang–Mills theory. It is shown that the Yang–Mills connection can be reconstructed from this parallel propagator.

The Yang–Mills equations are expressed as an equation for the parallel propagator. This equation can be given as a sum of two parts. The first of these, when set equal to zero on its own, satisfies the Huygens property and is soluble. When the second part is included, the Huygens property is destroyed. This leads to an approximation scheme which at first order is soluble yet already captures much of the non-linearity of Yang–Mills theory.

1. Introduction

In this note we wish to describe an alternate formulation of the standard Yang–Mills (Y–M) equations on Minkowski space, $\mathscr{M}$, in terms of a single matrix (or group) valued function on a six dimensional subspace of the space of paths. We will denote this function by G(path). More specifically this subspace is the space of null geodesics beginning at each point x^a of $\mathscr{M}$ and ending on future null infinity, $\mathscr{I}^+$. A natural parametrization for these paths are the coordinates x^a of the starting point of each path and the (complex) stereographic coordinates $(\zeta, \bar{\zeta})$ which label the sphere of generators of the future light-cone of x^a, where the path ends. A path is thus labeled by $(x^a, \zeta, \bar{\zeta})$. The function $G(x^a, \zeta, \bar{\zeta})$ is to be the parallel propagator, (with the Y–M connection, γ_a), of vectors in the fiber over x^a taken along the $(\zeta, \bar{\zeta})$ generator to the fiber over $\mathscr{I}^+$, i.e.

$$G(x^a, \zeta, \bar{\zeta}) = \mathscr{P} \exp\left(\int_x^\infty \gamma_a dx^a \right). \tag{1.1}$$

We will show that (1.1) can be inverted and thus the connection $\gamma_a(x)$ can be reconstructed (mod choice of gauge) from $G(x^a, \zeta, \bar{\zeta})$. From this fact, the Y–M field equations can be rewritten in terms of the G instead of the γ.