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q-Deformed Poincaré Algebra

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Abstract. The q-differential calculus for the q-Minkowski space is developed. The algebra of the q-derivatives with the q-Lorentz generators is found giving the q-deformation of the Poincaré algebra. The reality structure of the q-Poincaré algebra is given. The reality structure of the q-differentials is also found. The real Laplacian is constructed. Finally the comultiplication, counit and antipode for the q-Poincaré algebra are obtained making it a Hopf algebra.

1. Introduction

Quantum groups have already established themselves in such diverse branches of mathematics and theoretical physics as conformal field theory, integrable models, statistical mechanics, knot theory and topology of low-dimensional manifolds. Like many other notions (quantum mechanics, special relativity) quantum groups appear as some deformation of old "classical" objects, in this case groups. Although this type of deformation can be understood in terms of usual quantum mechanics, the idea of quantizing the symmetry itself is apparently new. The fruitfulness of this idea is supported by the number of geometric and algebraic notions which can be "q-deformed." First of all quantum groups can be viewed as symmetries of "quantum" spaces [1, 2]. Next the frame of differential calculus can be extended to include quantum groups and quantum spaces [3, 4].

The role of symmetry in physics is hard to overestimate. This explains the wide interest which quantum groups found among theoretical physicists. Particularly one is tempted to deform a real physical system in this spirit. This requires first of

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