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Smooth Conjugacy and S–R–B Measures for Uniformly and Non-Uniformly Hyperbolic Systems

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Abstract. We give a new proof of the fact that the eigenvalues at corresponding periodic orbits form a complete set of invariants for the smooth conjugacy of low dimensional Anosov systems. We also show that, if a homeomorphism conjugating two smooth low dimensional Anosov systems is absolutely continuous, then it is as smooth as the maps. We furthermore prove generalizations of these facts for non-uniformly hyperbolic systems as well as extensions and counterexamples in higher dimensions.

1. Introduction

The main purpose of this paper is to present a new proof of the following Theorem 1.1. The new methods we use allow us to make some generalizations, which have not appeared before, among them Theorem 1.2 and Theorem 1.3.

Theorem 1.1. *Let f, g be two $C^k, k = 2, 3, \ldots, \infty, \omega$ Anosov diffeomorphisms of a compact two dimensional manifold M (respectively σ_t, ψ_t two Anosov flows of a three dimensional manifold) and h a homeomorphism of M satisfying:*

$$h \circ f = g \circ h \tag{1.1}$$

(respectively $h \circ \sigma_t = \psi_t \circ h$). If the Lyapounov exponents at corresponding periodic orbits are the same, then $h \in C^{k-\varepsilon}$.

Theorem 1.2. *Let f, g (respectively σ_t, ψ_t), h be as in Theorem* 1.1 *and the manifold M be two (respectively three) dimensional. If h, h^{-1} are absolutely continuous with respect to Lebesgue measure then, h, h^{-1} are $C^{k-\varepsilon}$.*

Remark. We emphasize that Theorem 1.2 claims only that conjugacies which are both continuous and absolutely continuous are smooth. Since transitive Anosov systems preserving a smooth measure are Bernouilli, one could use Ornstein's theorem to produce absolutely continuous conjugacies between any two Anosov systems with the same metric entropy. They will, nevertheless be discontinuous in general.