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Deformations of Super Riemann Surfaces

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Abstract. Two different approaches to (Kostant-Leites-) super Riemann surfaces are investigated. In the local approach, i.e. glueing open superdomains by superconformal transition functions, deformations of the superconformal structure are discussed. On the other hand, the representation of compact super Riemann surfaces of genus greater than one as a fundamental domain in the Poincaré upper half-plane provides a simple description of super Laplace operators acting on automorphic p-forms.

Considering purely odd deformations of super Riemann surfaces, the number of linear independent holomorphic sections of arbitrary holomorphic line bundles will be shown to be independent of the odd moduli, leading to a simple proof of the Riemann–Roch theorem for compact super Riemann surfaces. As a further consequence, the explicit connections between determinants of super Laplacians and Selberg's super zeta functions can be determined, allowing to calculate at least the 2-loop contribution to the fermionic string partition function.

1. Introduction

In recent years, the theory of super Riemann surfaces has gained some attention, mainly motivated by the study of fermionic strings and superconformal field theories. As is well known, Polyakov's functional integral describing the g-loop contribution in the perturbation expansion of the fermionic string partition function can be reduced to a finite dimensional integral over super moduli space $\mathcal{M}_g$, the space parametrizing all super Riemann surfaces of genus g. The integrand contains some determinants of super Laplace operators acting on p-forms, which may be expressed via Selberg's super zeta functions in the case of genus greater than one. The explicit structure of these relations depends on the number of linear independent zero modes of the super Laplacian.

Two basically different approaches to supermanifolds exist: the one introduced by DeWitt [8] and the theory of graded manifolds in the sense of Kostant and Leites [27, 28]. I will follow the second approach, because it allows to use a lot of