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Geometric Quantization of the BRST Charge

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Abstract. In the first half of this paper (Sects. 1–4) we generalise the standard geometric quantization procedure to symplectic supermanifolds. In the second half (Sects. 5, 6) we apply this to two examples that exhibit classical BRST symmetry, i.e., we quantize the BRST charge and the ghost number. More precisely, in the first example we consider the reduced symplectic manifold obtained by symplectic reduction from a free group action with Ad*-equivariant moment map; in the second example we consider a foliated configuration space, whose cotangent bundle admits the construction of a BRST charge associated to this foliation. We show that the classical BRST symmetry can be described in terms of a hamiltonian supergroup action on the extended phase space, and that geometric quantization gives us a super-unitary representation of this supergroup. Finally we point out how these results are related to reduction at the quantum level, as compared with the reduction at the classical level.

1. Introduction and Summary

Recently there has been much interest at the classical level in the relation between reduced/constrained systems and the original (or extended) systems, especially with regard to the Poisson algebras. One considers a classical physical system described by a phase space (M_0, ω_0) (a symplectic manifold) subject to constraint functions $J_a = 0$, which we will assume to be first class. In an abstract way the actual phase space (also called the reduced phase space) is easy to describe. One considers the constraint set $C = \{m \in M_0 | \forall a \; J_a(m) = 0\}$ and the restriction of ω_0 to C. The leaves of the characteristic foliation $\mathscr{D}_{\text{char}}$ of $\omega_0|_C$ represent gauge equivalent points of this physical system and the quotient $M_r = C/\mathscr{D}_{\text{char}}$ gives us the actual phase space of the system. In favorable circumstances M_r is a (symplectic) manifold. If the constraint functions are derived from a proper symplectic group action on (M_0, ω_0), this is the well known Marsden Weinstein reduction [MW].

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