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Separation of Variables in the Classical Integrable $SL(3)$ Magnetic Chain

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Abstract. There are two fundamental problems studied by the theory of hamiltonian integrable systems: integration of equations of motion, and construction of action-angle variables. A third problem, however, should be added to the list: separation of variables. Though much simpler than the two others, it has important relations to quantum integrability. Separation of variables is constructed for the $SL(3)$ magnetic chain – an example of an integrable model associated to a nonhyperelliptic algebraic curve.

1. Introduction

Consider a completely integrable Hamiltonian system with D degrees of freedom. According to the definition of complete integrability due to Liouville-Arnold [1] it means that the system possesses exactly D independent Hamiltonians H_j commuting with respect to the Poisson bracket

$$\{H_j, H_k\} = 0, \qquad j, k = 1, \ldots, D. \tag{1}$$

There are three fundamental problems discussed in the theory of integrable systems. They are listed below in the order of decreasing complexity:

- Construction of action-angle variables.
- Integration of equations of motion.
- Separation of variables.

For the wide class of finite-dimensional integrable systems subject to the Inverse Spectral Transform Method an effective integration of equations of motion can be performed using the techniques of algebraic geometry [2]. As for the effective construction of the action-angle variables, it is a more difficult problem [3], especially

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