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On the Classification of Quasihomogeneous Functions

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Abstract. We give a criterion for the existence of a non-degenerate quasihomogeneous polynomial in a configuration, i.e. in the space of polynomials with a fixed set of weights, and clarify the relation of this criterion to the necessary condition derived from the formula for the Poincaré polynomial. We further prove finiteness of the number of configurations for a given value of the singularity index. For the value 3 of this index, which is of particular interest in string theory, a constructive version of this proof implies an algorithm for the calculation of all non-degenerate configurations.

1. Introduction

Recently, a particular class of singularities [1, 2], namely singularities of holomorphic quasihomogeneous functions, have been found useful for the classification of superconformal field theories (SCFT) with particular significance for the case of $N=2$ superconformal symmetry [3, 4] due to a non-renormalization theorem. The requirement of conformal invariance implies quasihomogeneity of degree 1 for the superpotential

$$W(\lambda^{n_i}\Phi_i)=\lambda^d W(\Phi_i) \tag{1}$$

in the effective Lagrangian description, with the scaling dimensions of the chiral superfields Φ_i translating into the weights $q_i=n_i/d$ of the variables X_i of a holomorphic function $W(X_i)$. The local algebra of this function, i.e. the quotient of the polynomial ring by the ideal generated by the gradients $\partial_j W(X_i)$, is isomorphic to the operator product algebra of chiral primary states [4]. In order that this

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