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Remarks on Duality of 1 Dimensional Quantum Spin Models

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Abstract. We present some results on duality maps and ground states of 1 dimensional quantum spin models. We also give some applications to Kramers Wannier duality and the nonlocal transformation that Kennedy and Tasaki discovered in their study of Haldane phase of quantum antiferromagnetic spin models.

1. Introduction

In this note, we consider certain nonlocal transformations which are used in the study of 1 dimensional quantum spin models. Examples are Kramers Wannier duality for quantum Potts models and the nonlocal transformation of Kennedy and Tasaki discovered in their study of Haldane phase of quantum antiferromagnetic spin models [5]. We will consider infinite volume ground states determined by these nonlocal transformations.

The ground state structure of quantum antiferromagnetic spin models attracted much attention of solid state physicists recently. Even in the 1 dimensional case, the characterization of phases is not yet fully understood. The isotropic antiferromagnetic Heisenberg model is believed to have a unique massless infinite volume ground state in the half odd integer spin case while Haldane's conjecture states that the model is massive in the integer spin (see [2]). However, it seems that no rigorous proof for uniqueness of the infinite volume ground state exists even in the spin 1/2 Heisenberg model which is a typical "solvable" model in the sense of Baxter ([2] deals with only the infinite volume limit under the free boundary condition while we expect that the infinite volume limit in *any boundary condition* converges to the same infinite volume state.) We believe that the rigorous study of infinite volume ground states is not merely of mathematical interest.

One nice feature of the 1 dimensional quantum spin model is existence of nonlocal transformations (e.g. bosonization, Kramers Wannier duality, etc.) that make some models formally equivalent to other simpler models. Such formal transformations give rise to "dual fields" (alternatively called disorder variables). The aim of this note