Commun. Math. Phys. 150, 59–64 (1992)

Communications in Mathematical Physics

Convergence of the Viscosity Method for a Nonstrictly Hyperbolic Conservation Law

Yunguang Lu

Wuhan Institute of Mathematical Sciences, Academia Sinica, P.O. Box 71007, Wuhan 430071, PRC

Received December 17, 1991; in revised form June 29, 1992

Abstract. A convergence theorem for the method of artificial viscosity applied to the nonstrictly hyperbolic system $v_t+(vu)_x=0$, $u_t+\left(\frac{1}{2}u^2+\int^v s(s+\delta)^{r-3}ds\right)_x=0$ ($\delta>0, r>3$) is established. Convergence of a subsequence in the strong topology is proved without uniform estimates on the derivatives using the theory of compensated compactness and an analysis of progressing entropy waves.

1. Introduction

In this paper we consider the existence of global weak solutions for nonlinear hyperbolic conservation laws

$$\begin{cases} v_t+(vu)_x=0\,, \\ u_t+\left(\frac{1}{2}u^2+\int^v s(s+\delta)^{r-3}ds\right)_x=0 \end{cases} \tag{1.1}$$

with initial data

$$(v(x,0),u(x,0))=(v_0(x),u_0(x))\,, \tag{1.2}$$

where δ, r are positive constants and $r>3$. When $\delta=0$, (1.1) is motivated by the isentropic equation of gas dynamics for a polytropic gas. The global weak solutions of which had been solved for the case of $1<r<3$ by using the Glimm difference scheme [1]. In the present paper, we shall study the system (1.1) with bounded measurable initial data (1.2) by using the established technique of compensated compactness given in [2, 3]. Through an analysis of progressing entropy waves, we establish a convergence theorem for the method of artificial viscosity applied to the system (1.1) and obtain the existence of the global weak solutions for the Cauchy problem (1.1), (1.2).