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Quantum $SU(2)$ and $E(2)$ Groups. Contraction Procedure

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Abstract. In [3] it was shown (in the framework of deformed enveloping algebras) that quantum $SU(2)$ and $E(2)$ groups are related by the contraction procedure. We consider the same problem on the C^*-level. As a result we find a number of formulae coupling the comultiplications in quantum $SU(2)$ and $E(2)$. In particular we show that the comultiplications in both groups are implemented by partial isometries. An unexpected feature of quantum $E(2)$ is discovered and the corresponding strange behavior of quantum $SU(2)$ is described.

0. Introduction

We shall consider two three-dimensional matrix groups:

$$SU(2) = \left\{ \begin{pmatrix} \alpha, & -\bar{\gamma} \\ \gamma, & \bar{\alpha} \end{pmatrix} \in M_{2\times 2}(\mathbf{C}) : |\alpha|^2 + |\gamma|^2 = \mathbf{1} \right\},$$

$$E(2) = \left\{ \begin{pmatrix} v, & n \\ 0, & \bar{v} \end{pmatrix} \in M_{2\times 2}(\mathbf{C}) : |\mathbf{v}| = \mathbf{1} \right\}.$$

They have the common subgroup S^1 consisting of all diagonal matrices. The corresponding homogeneous spaces are: the two-dimensional sphere in the case of $SU(2)$ and the two dimensional Euclidean plane in the case of $E(2)$. Since for small regions, the spherical geometry may be well approximated by the Euclidean one, we may expect that the two groups look very similar in a sufficiently small neighbourhood of S^1. To reveal this similarity we use the same coordinates to parametrize $SU(2)$ and $E(2)$.

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