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Calculation of General p-Adic Feynman Amplitude*

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Abstract. The general n-point massless p-adic Feynman amplitude with arbitrary parameters of analytic regularization for each line is calculated. This result is presented in the form of a sum over hierarchies of a given graph. The structure of ultraviolet and infrared divergences of p-adic Feynman amplitudes is characterized and the star-triangle uniqueness identity in the p-adic case is derived.

1. Introduction

In the past four years p-adic analysis [4, 15, 21] was applied in quantum theory [1, 7–10, 12, 13, 16–20, 22, 27, 28]. Several approaches were used. Within each of these approaches something was considered to be p-adic rather than real. For example, this can be the world sheet of a string (see, e.g., [7–10, 17]), or, our space-time itself [28]. In a paper by Lerner and Missarov [16] a generalized Koba-Nielsen amplitude was explicitly calculated and it was explained how an arbitrary one-dimensional p-adic massless Feynman amplitude can be computed.

The purpose of this work is to calculate the general p-adic massless Feynman amplitude with arbitrary parameters of analytic regularization in arbitrary space-time dimensions. The result will be written as a sum over hierarchies of the set of vertices of the given graph.

The paper is organized as follows. In the next section the main definitions of p-adic analysis are given and basic integrals are listed. In Sect. 3 auxiliary vacuum-type p-adic Feynman integrals are computed, and in Sect. 4 calculation of the general massless Feynman amplitude is presented. In Sect. 5 results of Sect. 4 are applied to simplest Feynman amplitudes. Furthermore, ultraviolet and infrared divergences of Feynman amplitudes are characterized and the star-triangle uniqueness identity is derived. Finally, in the Conclusion, the possibility of adelic formulae for massless Feynman amplitudes is discussed.

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