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On the Regularity of Solutions to the Yamabe Equation and the Existence of Smooth Hyperboloidal Initial Data for Einstein's Field Equations

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Abstract. The regularity of the solutions to the Yamabe Problem is considered in the case of conformally compact manifolds and negative scalar curvature. The existence of smooth hyperboloidal initial data for Einstein's field equations is demonstrated.

1. Introduction

In this paper we shall show the existence of smooth hyperboloidal initial data for Einstein's field equations. Such data occur under the following circumstances. Suppose one is given a solution to Einstein's source free field equations (of signature $(-,+,+,+)$) with cosmological constant $\Lambda \leq 0$ which is "asymptotically flat" respectively "asymptotically simple" in the sense of Penrose [15]. Let $\widetilde{M}$ be a space-like hypersurface in this space-time which extends up to the conformal boundary $\mathscr{I}$. The latter is a null hypersurface if $\Lambda = 0$ and time-like if $\Lambda < 0$. If one assumes that $\widetilde{M}$ can be extended across $\mathscr{I}$ as a smooth space-like hypersurface in some smooth conformal extension of the given space-time and denotes by ∂M the intersection of this extension with $\mathscr{I}$, then $M = \widetilde{M} \cap \partial M$ will be a smooth compact manifold with boundary.

We call the triple $(\widetilde{M}, \tilde{g}, \tilde{\chi})$, where $\tilde{g}$ is the Riemannian metric and $\tilde{\chi}$ the second fundamental form induced on $\widetilde{M}$ by the space-time metric, a "hyperboloidal initial data set." Since this initial data set has been obtained from a solution to the Einstein equations the fields $\tilde{g}$ and $\tilde{\chi}$ satisfy the constraint equations. Because of the specific geometric situation which is represented by this data set and because it has been derived from a space-time admitting a smooth conformal boundary $\mathscr{I}$ at infinity the fields $\tilde{g}$ and $\tilde{\chi}$ satisfy certain special smoothness respectively fall-off conditions at ∂M.

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