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Weak Disorder Expansions for the Anderson Model on a One Dimensional Strip at the Center of the Band

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Abstract. We study the asymptotic behavior of the averaged diagonal matrix elements of the Greens kernel for the Anderson Model on a one-dimensional strip and for a set of special energies close to the center of the band.

I. Introduction

Let ℓ be a positive integer and let $\mathscr{D}_\ell$ be the one dimensional lattice strip of with ℓ, i.e., $\mathscr{D}_\ell = \mathbb{Z} \times \{1, \ldots, \ell\}$, where $\mathbb{Z}$ is the set of all integers.

The Anderson model [1] on $\mathscr{D}_\ell$ is given by the random Hamiltonian $H_\lambda = -\frac{1}{2}\Delta + \lambda V$ on $\ell^2(\mathscr{D}_\ell)$, where

$$(\Delta u)(x) \sum_{y \in \mathscr{D}_\ell} (\Delta)_{x,y} u(y)$$

with

$$(\Delta)_{x,y} = \begin{cases} 1 & \text{if } x - y \in \{(0,1), (-1,0), (1,0), (0,-1)\} \\ 0 & \text{otherwise}, \end{cases}$$

and

$$(Vu)(x) = V(x)\, u(x),$$

where $\{V(x)\}_{x \in \mathscr{D}_\ell}$ are i.i.d. real random variables with common distribution μ whose characteristic function will be denoted by h and λ is a real number.

Let m be a positive integer, Λ^m be the discrete rectangle $[-m, m] \times \{1, 2, \ldots, \ell\}$ and $H_{m,\lambda}$ denote the H_λ restricted to $\ell^2(\Lambda_\ell)$ with boundary conditions $u(x) = 0$ for all $x \notin \Lambda^m$. Let $x, y \in \mathscr{D}_\ell$, $\eta > 0$ and let $\delta_x, \delta_y \in \ell^2(\mathscr{D}_\ell)$ be the delta functions at the points x and y respectively. We shall use the notations

$$G_m^\lambda(x, y, E + i\eta) = \left\langle \delta_x \left| \frac{1}{H_{m,\lambda} - E - i\eta} \right| \delta_y \right\rangle,$$

$$J_\lambda^j(E + i\eta) = \lim_{m \to +\infty} \mathbb{E}\{G_m^\lambda((0,j), (0,j), E + i\eta)\},$$

where $j = 1, \ldots, \ell$ and $E\{\cdot\}$ denotes the expectation with respect to the disorder.