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Action of Truncated Quantum Groups on Quasi-Quantum Planes and a Quasi-Associative Differential Geometry and Calculus

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Abstract. If q is a p^{th} root of unity there exists a quasi-coassociative truncated quantum group algebra whose indecomposable representations are the physical representations of $U_q(sl_2)$, whose coproduct yields the truncated tensor product of physical representations of $U_q(sl_2)$, and whose R-matrix satisfies quasi-Yang Baxter equations. These truncated quantum group algebras are examples of weak quasitriangular quasi-Hopf algebras ("quasi-quantum group algebras") $\mathcal{G}^*$. We describe a space $\mathcal{F}^T$ of "functions on the quasi quantum plane," i.e. of polynomials in noncommuting complex coordinate functions z_a, on which multiplication operators Z_a and the elements of $\mathcal{G}^*$ can act, so that z_a will transform according to some representation τ^f of $\mathcal{G}^*$. $\mathcal{F}^T$ can be made into a quasi-associative graded algebra $\mathcal{F}^T = \bigoplus_{n>0} \mathcal{F}^{T(n)}$ on which elements of $\mathcal{G}^*$ act as generalized derivations. In the special case of the truncated $U_q(sl_2)$ algebra we show that the subspaces $\mathcal{F}^{T(n)}$ of monomials in z_a of n^{th} degree vanish for $n \geq p-1$, and that $\mathcal{F}^{T(n)}$ carries the $2J+1$ dimensional irreducible representation of $\mathcal{G}^*$ if $n = 2J$, $J = 0, \frac{1}{2}, \ldots, \frac{1}{2}(p-2)$. Assuming that the representation τ^f of the quasi-quantum group algebra gives rise to an R-matrix with two eigenvalues, we develop a quasi-associative differential calculus on $\mathcal{F}^T$. This implies construction of an exterior differentiation, a graded algebra $\Lambda\mathcal{F}^T = \bigoplus \Lambda^n\mathcal{F}^T$ of forms and partial derivatives. A quasi-associative generalization of noncommutative differential geometry is introduced by defining a covariant exterior differentiation of forms. It is covariant under $\mathcal{G}^*$-valued gauge transformations.

0. Introduction

To explain the problem which we address, we recall the theory of the complex quantum plane [1, 2, 3].

The algebra $\mathcal{F}$ of polynomial functions on the quantum plane is a noncommutative but associative deformation of the commutative algebra $\mathcal{F}_{cl}$ of polynomial functions

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