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Perturbation Theory for Kinks

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Abstract. In this paper we prove the validity of formal asymptotic results on perturbation theory for kink solutions of the sine-Gordon equation, originally obtained by McLaughlin and Scott. We prove that for appropriate perturbations, of size ε in an appropriate norm, slowly varying in time in the rest frame of the kink, the shape of the kink is unaltered in the L^∞ norm to $O(\varepsilon)$ for a time of $O\left(\frac{1}{\varepsilon}\right)$. The kink parameters, which represent its velocity and centre, evolve slowly in time in the way predicted by the asymptotics. The method of proof uses an orthogonal decomposition of the solution into an oscillatory part and a one-dimensional "zero-mode" term. The slow evolution of the kink parameters is chosen so as to suppress secular evolution of the zero-mode.

Section 1. Introduction and Statement of Results

In this paper we prove the validity of formal asymptotic results due to McLaughlin and Scott (1978) and Karpman and Solov'ev (1981) for appropriate nonlinear perturbations of the sine-Gordon equation:

$$\theta_{TT} - \theta_{XX} + \sin\theta + \varepsilon g = 0\,. \tag{1.1}$$

More precisely, we prove the existence, for long but finite times, of solutions to this equation which approximate travelling waves of the unperturbed equation, with parameters evolving slowly in time under the action of the perturbation εg. The travelling waves of interest are uniformly moving kinks. Kinks are members of a two parameter family of solutions of the unperturbed equation:

$$\theta_{TT} - \theta_{XX} + \sin\theta = 0 \tag{1.2}$$

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