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Hidden Quantum Groups Inside Kac-Moody Algebra

A. Alekseev, L. Faddeev, and M. Semenov-Tian-Shansky

St. Petersburg Branch of Steklov Mathematical Institute, Fontanka 27,
St. Petersburg 191011, Russia

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Abstract. A lattice analogue of the Kac-Moody algebra is constructed. It is shown that the generators of the quantum algebra with the deformation parameter $q=\exp(i\pi/k+h)$ can be constructed in terms of generators of the lattice Kac-Moody algebra (LKM) with the central charge k. It appears that there exists a natural correspondence between representations of the LKM algebra and the finite dimensional quantum group. The tensor product for representations of the LKM algebra and the finite dimensional quantum algebra is suggested.

1. Introduction

Fascinating links between conformal field theory and quantum groups discovered recently suggest that quantum groups also have a direct bearing on the representation theory of Kac-Moody algebras. It is the purpose of the present note to trace down this hidden quantum group symmetry in the framework of Kac-Moody algebras. Our main result is that the monodromy of quantum Kac-Moody current when properly regularized satisfies the commutation relations of the quantized universal enveloping algebra $U_q(\mathfrak{g})$ with q related to the central charge k via $q=\exp\left(\frac{\pi i}{k+n}\right)$. The regularized definition of the monodromy is based in its turn on a lattice version of the current algebra which we also describe in this paper. This algebra associated with a periodic 1-dimensional lattice is already quantum (i.e. incorporates parameter q; in fact, it is defined for any q, not only for roots of unity) and also takes into account the central charge. It may be regarded as a nontrivial deformation of $U_q(\mathfrak{g})^{\otimes N}$. (The very existence of such deformations is a typically quantum phenomenon. Indeed, it is well known that classical semi-simple Lie groups and Lie algebras are rigid. By contrast, quantum universal enveloping algebras admit certain deformations which may be regarded as finite-dimensional counterparts of central extensions of current algebras.) Our first key result is the monodromy theorem for this lattice algebra which asserts that the