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Special Geometry, Cubic Polynomials and Homogeneous Quaternionic Spaces

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Abstract. The existing classification of homogeneous quaternionic spaces is not complete. We study these spaces in the context of certain $N = 2$ supergravity theories, where dimensional reduction induces a mapping between *special* real, Kähler and quaternionic spaces. The geometry of the real spaces is encoded in cubic polynomials, those of the Kähler and quaternionic manifolds in homogeneous holomorphic functions of second degree. We classify all cubic polynomials that have an invariance group that acts transitively on the real manifold. The corresponding Kähler and quaternionic manifolds are then homogeneous. We find that they lead to a well-defined subset of the normal quaternionic spaces classified by Alekseevskiĭ (and the corresponding special Kähler spaces given by Cecotti), but there is a new class of rank-3 spaces of quaternionic dimension larger than 3. We also point out that some of the rank-4 Alekseevskiĭ spaces were not fully specified and correspond to a finite variety of inequivalent spaces. A simpler version of the equation that underlies the classification of this paper also emerges in the context of W_3 algebras.

1. Introduction

Supersymmetric field theories in a variety of space-time dimensions give rise to non-linear sigma models with a restricted target-space geometry. There are many examples in the literature where this phenomenon led to surprising results, sometimes with interesting connections to mathematics. Furthermore, the fact that some of these supersymmetric theories in different space-time dimensions are related by (supersymmetric) dimensional reduction offers a way of connecting seemingly unrelated geometries.

In the context of this paper $N = 2$ supergravity is relevant. In five space-time dimensions, one may consider the coupling of a certain number (say $n - 1$) of supersymmetric abelian vector multiplets. As was shown some time ago [1], these

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