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Solving the KP Hierarchy by Gauge Transformations

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Abstract. We show that it is convenient to use "gauge" transformations (sometimes called explicit Bäcklund transformations) to generate new solutions for the KP hierarchy. Two particular kinds of gauge transformation operators, constructed out of the initial wave functions, are of fundamental importance in this approach. Through such gauge transformations, a very simple formula for the tau-function is obtained, encompassing and unifying all kinds of existing solutions. The corresponding free fermion representation and Baker functions for the new τ function can also be constructed.

1. Introduction

There are several different ways to formulate the mathematical problem of the KP hierarchy equations. For our purpose it is most convenient to adopt the pseudo-differential operator formalism developed by Sato and his school [1–5]. By the KP hierarchy we mean a particular infinite set of coupled nonlinear equations for u_i ($i = 2, 3, \ldots$), where each $u_i = u_i(x_1, x_2, x_3, \ldots)$ depends on one "spatial" variable x_1 and infinitely many "time" variables $x_2, x_3, \ldots$. These coupled equations are to be generated in the following way [2].

Let Λ denote the pseudo-differential operator

$$\Lambda \equiv \partial + u_2\partial^{-1} + u_3\partial^{-2} + u_4\partial^{-3} + \ldots, \tag{1.1}$$

where $\partial \equiv \partial/\partial x_1$, and ∂^{-1} is a suitable inverse of ∂, obeying the generalized Leibniz rule

$$\partial^{-n} \circ f(x_1) = \sum_{l=0}^{\infty} (-1)^l \frac{(n+l-1)!}{l!(n-1)!} f^{(l)}(x_1)\partial^{-n-l}, \quad (n > 0). \tag{1.2}$$

For an operator multiplication we put a "$\circ$" in between, e.g., $\partial \circ f \equiv \partial f + f\partial\,\circ$. Now let

$$B_n \equiv [\Lambda^n]_+ , \tag{1.3}$$