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Tensor Products of Quantized Tilting Modules

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Abstract. Let U_k denote the quantized enveloping algebra corresponding to a finite dimensional simple complex Lie algebra $\mathfrak{L}$. Assume that the quantum parameter is a root of unity in k of order at least the Coxeter number for $\mathfrak{L}$. Also assume that this order is odd and not divisible by 3 if type G_2 occurs. We demonstrate how one can define a reduced tensor product on the family $\mathscr{F}$ consisting of those finite dimensional simple U_k-modules which are deformations of simple $\mathfrak{L}$-modules and which have non-zero quantum dimension. This together with the work of Reshetikhin–Turaev and Turaev–Wenzl prove that $(U_k, \mathscr{F})$ is a modular Hopf algebra and hence produces invariants of 3-manifolds. Also by recent work of Duurhus, Jakobsen and Nest it leads to a general topological quantum field theory. The method of proof explores quantized analogues of tilting modules for algebraic groups.

It was recently shown by Reshetikhin and Turaev [RT] that one can obtain invariants of 3-manifolds via quantum groups (see also [KM] and [TW]). In fact, [RT] contains a general procedure for the construction of invariants from modular Hopf algebras, and it was proved that the quantum group for sl_2 is a modular Hopf algebra. Moreover, in [TW] it is proved that the quantum groups corresponding to root systems of classical types are quasi-modular Hopf algebras and that such an algebra similarly produces invariants of 3-manifolds.

The purpose of this paper is to prove that a quantum group corresponding to any finite root system is a modular Hopf algebra (actually we only check two of the axioms since the remaining ones are verified in [TW]). More specifically, we prove this over any field of characteristic zero in which the quantum parameter is a root of unity of odd order l bigger than the Coxeter number for the root system in question (if the root system involves type G_2, we assume l also to be prime to 3).

In another recent (and related) development Durhuus, Jakobsen and Nest, [DJN], have demonstrated (generalising the results of Turaev and Viro [TV]) how one may obtain a topological quantum field theory from the rather general setup of an associative algebra with a distinguished finite set of irreducible representations