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Spectral Theory, Zeta Functions and the Distribution of Periodic Points for Collet–Eckmann Maps

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Abstract. We study unimodal interval maps T with negative Schwarzian derivative satisfying the Collet–Eckmann condition $|DT^n(Tc)| \geqq K\lambda_c^n$ for some constants $K > 0$ and $\lambda_c > 1$ (c is the critical point of T). We prove exponential mixing properties of the unique invariant probability density of T, describe the long term behaviour of typical (in the sense of Lebesgue measure) trajectories by Central Limit and Large Deviations Theorems for partial sum processes of the form $S_n = \sum_{i=0}^{n-1} f(T^i x)$, and study the distribution of "typical" periodic orbits, also in the sense of a Central Limit Theorem and a Large Deviations Theorem.

This is achieved by proving quasicompactness of the Perron Frobenius operator and of similar transfer operators for the Markov extension of T and relating the isolated eigenvalues of these operators to the poles of the corresponding Ruelle zeta functions.

1. Introduction

During the last years considerable progress was made towards the understanding of the metric structure of general unimodal maps with negative Schwarzian derivative (henceforth called S-unimodal maps). The likely limit set in the sense of Milnor [Mi] was described and related to the conservativeness/transitiveness of the map with respect to Lebesgue measure [BL1, BL3, GJ, Ma, K4]. The ergodicity of S-unimodal maps without stable periodic orbit was proved in [BL2, BL3, Ma]. (For a discussion of these results see [HK3].) Also new sufficient or equivalent conditions for the existence of invariant probability densities were found. (The uniqueness of such invariant densities follows from the ergodicity of T.) Before we describe some of these results, we introduce the class of maps we are going to investigate:

T: $[0, 1] \to [0, 1]$ is of class C^3 and has a unique nondegenerate critical point c of order l, see (1.2).

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