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Regularity of Harmonic Maps with Prescribed Singularities

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Abstract. In this paper, we studied the regularity problem for harmonic maps into hyperbolic spaces with prescribed singularities along codimension two submanifolds. This is motivated from one of Hawking's conjectures on the uniqueness of Kerr solutions among all axially symmetric asymptotically flat stationary solutions to the vacuum Einstein equation in general relativity.

1. Introduction

In the last three decades, much progress has been made on harmonic maps between Riemannian manifolds. Among the outstanding ones, for instance, are the existence of Eells and Sampson [ES] on harmonic maps into nonpositively curved manifolds, with the generalization of R. Hamilton [Ha] to manifolds with boundary, the ones of Sacks and Uhlenbeck [SU], Lemaire [Le] and R. Schoen and S.-T. Yau on harmonic maps defined on Riemann surfaces, and regularity theories of R. Schoen and K. Uhlenbeck [SU1, SU2]. Prior to [SU1, SU2] there had been some regularity theorems due to Hildebrandt, Giusti, Giaquinta (see for example [Gi]) under various assumptions on the target manifolds. These results have brought tremendous new understandings of the geometry of manifolds.

In this paper, we consider the following problem. Let (M, ds^2) be a n-dimensional complete Riemannian manifold with or without boundary, and $N \subset M$ be a codimension two closed submanifold; let h be a smooth map from $M \backslash N$ into the naturally compactified hyperbolic space $\bar{H}^m$ such that $h(M \backslash N) \subset H^m$, where H^m is upper-half-space model of m-hyperbolic space form. Then we would like to find a harmonic map from $M \backslash N$ in H^m with "similar" asymptotic behavior to h along N. One natural approach is to perturb h to obtain the harmonic one. To make it

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