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$U(1) \times SU(2)$-Gauge Invariance of Non-Relativistic Quantum Mechanics, and Generalized Hall Effects

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Abstract. We show that the non-relativistic quantum mechanics of particles with spin coupled to an electromagnetic field has a natural $U(1) \times SU(2)$ gauge invariance. Ward identities reflecting this gauge invariance combined with an assumption of incompressibility of a system of such particles in an appropriate external field and for suitable values of the particle density permit us to determine the form of the effective action of the system as a functional of small fluctuations in the electromagnetic field, in the large-distance-, adiabatic limit. In this limit, the action is found to have a universal form. We present explicit results for two-dimensional, incompressible electron fluids and apply them to derive the equations of linear response theory, describing a variety of generalized Hall effects. Sum rules for the Hall conductivities, magnetic susceptibilities and other quantities of physical interest are found.

1. Introduction and Summary of Main Results

In this paper we study the physics of two-dimensional (2d) electronic and magnetic systems, e.g., of heterojunctures or 2d chiral spin liquids. Such systems are described in theories of the quantized Hall effect or of layered superconductors.

A basic recent observation is that the large-scale, low-frequency physics of incompressible electron fluids exhibits *universal features*. Incompressibility is understood as the absence of dissipative processes. Experimentally, it corresponds to a vanishing longitudinal resistance, i.e., $R_L = 0$. For incompressible electron fluids one can identify interesting physical quantities, such as the Hall conductivity or the quantum numbers of excitations above the groundstate, which only depend on the large-scale, low-frequency properties of the system and which can therefore be predicted precisely *without* detailed knowledge of the microscopic dynamics. A related notion of universality is familiar from the theory of critical phenomena accompanying continuous phase transitions. The idea that incompressible quantum fluids exhibit universal large-scale, low-frequency behaviour plays an important role in the analysis of the quantum Hall effect reported in [1–3].