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Relativistic Invariance and Charge Conjugation in Quantum Field Theory

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Abstract. We prove that superselection sectors with finite statistics in the sense of Doplicher, Haag, and Roberts are automatically Poincaré covariant under natural conditions (e.g. split property for space-like cones and duality for contractible causally complete regions). The same holds for topological charges, namely sectors localized in space-like cones, providing a converse to a theorem of Buchholz and Fredenhagen. We introduce the notion of weak conjugate sector that turns out to be equivalent to the DHR conjugate in finite statistics. The weak conjugate sector is given by an explicit formula that relates it to the PCT symmetry in a Wightman theory. Every Euclidean covariant sector (possibly with infinite statistics) has a weak conjugate sector and the converse is true under the above natural conditions. On the same basis, translation covariance is equivalent to the property that sectors are sheaf maps modulo inner automorphisms, for a certain sheaf structure given by the local algebras. The construction of the weak conjugate sector also applies to the case of local algebras on S^1 in conformal theories. Our main tools are the Bisognano-Wichmann description of the modular structure of the von Neumann algebras associated with wedge regions in the vacuum sector and the relation between Jones index theory for subfactors and the statistics of superselection sectors.

1. Introduction

According to the Haag-Kastler approach to Quantum Field Theory [19], the physical content of the theory is encoded in the net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ of local observables associated with double cones of the Minkowski space. In this framework the physical representations (the superselection sectors or quantum charges introduced in [38]) are to correspond to a family of (unitary equivalence class of) representations of the quasi-local C^*-algebra $\mathcal{A} = \bigcup \mathcal{A}(\mathcal{O})^-$. Since not all

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