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Large Time Behavior of Classical N-body Systems

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Abstract. Asymptotic properties of solutions of N-body classical equations of motion are studied.

1. Introduction

A system of N classical particles interacting with pair potentials can be described with a Hamiltonian of the form

$$H = \sum_{i=2}^{N} \frac{1}{2m_i} \xi_i^2 + \sum_{i>j=1}^{N} V_{ij}(x_i - x_j) \tag{1.1}$$

defined on the phase space $X \times X'$, where $X = \mathbb{R}^{3N}$ and X' is its conjugate space. Following Agmon [A] it has become almost standard in the mathematically oriented literature to replace (1.1) with an essentially more general class of Hamiltonians, sometimes called generalized N-body Hamiltonians. They are functions on $X \times X'$ of the form

$$H = \frac{1}{2}\xi^2 + \sum_{a \in \mathscr{A}} V^a(x^a), \tag{1.2}$$

where X is a Euclidean space, $\{X^a : a \in \mathscr{A}\}$ is a family of subspaces closed wrt the algebraic sum and containing $\{0\}$, and x^a denotes the orthogonal projection of x onto X^a. It is easy to see that after a change of coordinates any Hamiltonian of the form (1.1) belongs to the class (1.2).

Typical assumptions imposed in the literature on the potential are

$$|\partial^\alpha V^a(x^a)| < c_\alpha \langle x^a \rangle^{-\mu-|\alpha|}, \tag{1.3}$$

where $\mu > 0$. If $\mu > 1$ then we say that the potentials are short range, otherwise they are long range. Note that (1.2) has an obvious quantum analog, which is the

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