

The Index of the Scattering Operator on the Positive Spectral Subspace

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Abstract. We construct the scattering operator for a spinor field in a time dependent background by the Dyson expansion. Then we show that the restriction of the scattering operator to the positive spectral subspace (with respect to a reference Hamiltonian) is Fredholm. The computation of the index of this restriction is reduced to the index computation for an elliptic pseudodifferential operator of order zero. We obtain the index in terms of a cohomological formula by means of the Atiyah-Singer index theorem.

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1. Introduction

The scattering operator Ω_- describes how the time evolution of a field governed by a time dependent Hamiltonian $H(t)$ behaves in comparison with an evolution given by a constant reference Hamiltonian H_0 . The operator Ω_- maps the space of incoming states with respect to H_0 to the incoming states of $H(t)$. $\Omega_- \phi =: \psi$ is