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# Perturbative Renormalization of Composite Operators via Flow Equations I

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**Abstract.** We apply the general framework of the continuous renormalization group, whose significance for perturbative quantum field theories was recognized by Polchinski, to investigate by new and mathematically simple methods the perturbative renormalization of composite operators. In this paper we demonstrate the perturbative renormalizability of the Green functions of the Euclidean massive $\Phi_4^4$ theory with one insertion of a (possibly oversubtracted, in the BPHZ language) composite operator. Moreover we show that our method admits an easy proof of the Zimmermann identities and of the Lowenstein rule.

## 1. Introduction

In the framework of the BPHZ renormalization theory the first systematic definition and study of renormalized composite operators has been performed by Zimmermann [1, 2, 3]. He introduced a special set of composite operators which can be interpreted as generalized Wick normal products and which ever since have been called Zimmermann normal products [1, 2]. One of the remarkable features of these Zimmermann normal products is that they are a convenient tool to prove the validity of Wilson's short distance expansion in renormalized perturbation theory [1, 3]. Moreover, Zimmermann showed [1, 2] that these normal products obey a set of linear relations, the Zimmermann identities, which turned out to be especially well suited for analyzing some of the main structural properties of perturbative Green functions; the importance of the Zimmermann identities can hardly be overemphasized. For instance, Lowenstein and his collaborators (mostly) have employed Zimmermann's normal product techniques and identities in conjunction with Lowenstein's rule [4] (another identity which is obeyed by the Zimmermann normal products) to construct renormalized symmetry generators (e.g. an energy momentum tensor for $\Phi_4^4$ [4]), to search for broken symmetries (e.g. asymptotic scale invariance in $\Phi_4^4$ [4, 5, 6] and in the 2-d massive Thirring

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