Commun. Math. Phys. 148, 209–216 (1992)

Communications in
Mathematical
Physics

Lines in Space-Times

J.-H. Eschenburg[1] **and G. J. Galloway**[2,★]

[1] Institut für Mathematik, Universität Augsburg, Universitätsstraße 8, W-8900 Augsburg, FRG
[2] Department of Mathematics and Computer Sciences, P.O. Box 249085, Coral Gables, FL 33124, USA

Received August 29, 1991; in revised form March 17, 1992

Abstract. We construct a complete timelike maximal geodesic ("line") in a timelike geodesically complete spacetime M containing a compact acausal spacelike hypersurface S which lies in the past of some S-ray. An S-ray is a future complete geodesic starting on S which maximizes Lorentzian distance from S to any of its points. If the timelike convergence condition (strong energy condition) holds, a line exists only if M is static, i.e. it splits geometrically as space $\times$ time. So timelike completeness must fail for a nonstatic spacetime with strong energy condition which contains a "closed universe" S with the above properties.

1. Introduction

Let M be a timelike geodesically complete time-oriented Lorentzian manifold containing a compact spacelike acausal hypersurface S. A conjecture stated by R. Bartnik [B] says: If M satisfies the timelike convergence condition (strong energy condition), then M splits isometrically as space $\times$ time. (In fact, Bartnik assumes S to be a Cauchy hypersurface.) By the Lorentzian splitting theorem [N], this statement is true if we can construct a timelike line, i.e. an inextendible maximal timelike geodesic. However, without the timelike convergence condition, such a line need not exist (cf. [EG]). It is the aim of the present paper to construct a timelike line if S lies in the past of some S-ray, i.e. a future inextendible causal curve γ starting on S such that $\gamma \mid [0, t]$ is a curve of maximal length between S and $\gamma(t)$ for all $t > 0$.

The main results are stated and proved in Sect. 5; the ingredients are given in Sects. 2–4. For standard facts in Lorentzian geometry and for standard notation (such as I^+, J^+, D^+, H^+) we refer to [HE, BE].

★ Partially supported by NSF grant no. DMS-9006678