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Equivalence of Gibbs and Equilibrium States for Homeomorphisms Satisfying Expansiveness and Specification

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Dedicated to Joel Lebowitz

Abstract. Let M be a compact metrizable space, f: $M \to M$ a homeomorphism satisfying expansiveness and specification, and A: $M \to \mathbb{R}$ a function such that

$$\left| \sum_{k=0}^{n-1} [A(f^k x) - A(f^k y)] \right| \leqq K(\varepsilon) < \infty$$

whenever $n \geqq 1$ and x, y are (ε, n)-close (i.e. $d(f^k x, f^k y) < \varepsilon$ for $k = 0, \ldots, n-1$, some fixed choice of metric d and expansive constant $\varepsilon > 0$). Under these conditions, Bowen has shown that there is a unique *equilibrium state* ρ for A. Assuming that $K(\delta) \to 0$ when $\delta \to 0$, we show that ρ is also the unique *Gibbs state* for A. We further define *quasi-Gibbs states* and show that ρ is the unique f-invariant quasi-Gibbs state for A.

0. Introduction

The concepts of equilibrium state and of Gibbs state come from the statistical mechanics of (spatially) infinite systems. States of thermal equilibrium of such systems can be defined either *globally* by a *variational principle* (this gives *equilibrium states* or locally by specifying certain conditional probabilities (this gives *Gibbs states*). Under fairly general conditions one can prove that equilibrium states coincide with translationally invariant Gibbs states (Dobrushin [4, 5], Lanford and Ruelle [8]). For one-dimensional statistical mechanics (with a natural *mixing* condition and *short range interactions*) Gibbs states are automatically translationally invariant, hence equivalent to equilibrium states; there is in fact one equilibrium state (i.e. these systems have no *phase transitions*).

The invariance under translations for (one-dimensional) statistical mechanics is a special example of invariance under a homeomorphism f of a compact metrizable space M. It is natural to try to extend the theory of equilibrium and Gibbs states to this more general situation. For equilibrium states, this is relatively easy

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