Commun. Math. Phys. 148, 101–116 (1992)

Communications in Mathematical Physics

Explicit Solution for p–q Duality in Two-Dimensional Quantum Gravity

Masafumi Fukuma[1]*, Hikaru Kawai[1] and Ryuichi Nakayama[2]*****

[1] Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan
[2] National Laboratory for High Energy Physics (KEK), Tsukuba-shi, Ibaraki 305, Japan

Received May 23, 1991

Abstract. By using Sato's infinite dimensional Grassmannian theory of the KP hierarchy, we study the global structure of the theory space of 2D quantum gravity coupled to various minimal conformal fields labeled by a pair of integers (p, q). After giving a rigorous proof of the equivalence of Douglas's equation and the Schwinger–Dyson equation (W-constraint on a τ function), we establish the p–q duality of the (p, q) quantum gravity at Green's function level. As an application, we discuss the metamorphosis of operators under unitarity-preserving renormalization group flows.

1. Introduction

Recent developments in two-dimensional (2D) quantum gravity have revealed a great deal of its mathematical and physical structures [1]. However, our understanding of the theory has not yet reached a stage that we can extract enough information for constructing, for example, a general framework for higher dimensional gravities. In order to make further developments in this direction, it is essential to find a universal formulation that describes the operator structures of the whole theory space of 2D gravity. At present, we have two promising formulations; one is based on Douglas's equation [2] and the other on the Schwinger–Dyson (S–D) equation [3, 4] (and an equivalent approach based on the action principle [5, 6]). Either of them, however, is not completely satisfactory, but they are complementary to each other in the following sense. The former is suitable for describing the theory space but not convenient for examining the relations among various operators. On the other hand the latter makes the operator structures manifest but does not give a completely universal description of the whole theory space. In fact, in the latter formulation 2D gravities coupled to (p, q) conformal

* Address after September 1, 1992: Newman Laboratory, Cornell University, Ithaca, NY 14853-5001, USA. E-mail address: tkyvax$hepnet::fukuma
** E-mail address: tkyvax$hepnet::kawai
*** E-mail address: nakayama@jpnkekvm.bitnet, nakayama@kekvax.kek.jp