

# Complex Equivariant Intersection, Excess Normal Bundles and Bott-Chern Currents

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*This paper is dedicated to Professor P. Malliavin*

**Abstract.** The purpose of this paper is to establish an intersection formula in equivariant complex geometry, in the presence of an excess normal bundle. The contribution of the excess normal bundle to the formula appears through an additive genus  ${}^K R$ . In a forthcoming paper, an infinite dimensional analogue of this formula will be shown to be the result of Bismut-Lebeau on the behaviour of Quillen metrics under complex immersions.

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