

Complex Quantum Groups and Their Quantum Enveloping Algebras

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Abstract. We construct complexified versions of the quantum groups associated with the Lie algebras of type A_{n-1} , B_n , C_n , and D_n . Following the ideas of Faddeev, Reshetikhin and Takhtajan we obtain the Hopf algebras of regular functionals $U_{\mathcal{R}}$ on these complexified quantum groups. In the special example A_1 we derive the q -deformed enveloping algebra $U_q(sl(2, \mathbb{C}))$. In the limit $q \rightarrow 1$ the undeformed $U(sl(2, \mathbb{C}))$ is recovered.

1. Introduction

For quantum groups associated with the Lie algebras g of type A_{n-1} , B_n , C_n , and D_n there exist well defined correlations between the quantum group itself and the corresponding q -deformed universal enveloping algebra $U_q(g)$ [Dri, FRT]. Coming from the quantum group, one can construct the algebra of regular functionals which is shown to be the algebra $U_q(g)$ for a certain completion. Though the q -deformed Lorentz group already exists in at least two versions [CSSW, PW], there is not yet such a straightforward procedure like in the case of compact Lie groups to derive the corresponding quantized universal enveloping algebra. However this q -deformed algebra is the very object of interest since it should be fundamental for the construction of a q -deformed relativistic field theory.

In this paper we present the quantized universal enveloping algebra $U_q(sl(2, \mathbb{C}))$ of the q -deformed Lorentz group $Sl_q(2, \mathbb{C})$. In Sect. (2) we construct complex quantum groups for the Lie algebras A_{n-1} , B_n , C_n , and D_n . These are complexifications of the original quantum groups. The algebraic relations can be written in a generalized $R\overline{T\overline{T}}$ -formulation and the usual determinant or metric relations. Following the ideas of [FRT] this fact is used in Sect. (3) to build up the algebra of regular functionals on the complex quantum groups¹. The approach in this paper is purely algebraic without

¹ The same universal enveloping algebra corresponding to the complex quantum group is constructed by analyzing the algebra of the fundamental bicovariant bicomodule [CW]